Can Self-focusing of Intense Laser Beams Cause Bose-Einstein Condensation into a Superfluid State?

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The cause of the phenomenon of self-focusing of intense laser radiation in solids is the nonlinear intensity dependent refractive index [1]

$$n = n_1 + n_2 \, \overline{E^2} \,, \tag{1}$$

where n_1 is the normal refractive index and $\overline{E^2}$ the time average of the electric field of the laser beam radiation. The coefficient n_2 determines the magnitude of the nonlinear behavior of the refractive index. Self-focusing happens provided the laser beam power P exceeds a critical value P_c, which is [1] (in cgs units)

$$P_c = c^3/4 \, n_2 \, \omega^2 \,. \tag{2}$$

Typically, one may have $n_2 \cong 10^{-11}$ [cgs], $\lambda \sim 10^{-4}$ cm ($\omega \sim 2 \times 10^{15}$ sec^{-1}), with the result that $P_c \sim 2 \times 10^{11}$ erg/sec $= 2 \times 10^4$ Watt. Therefore, if $P > P_c$ the beam begins to undergo self-focusing. The critical temperature below which this is going to happen is a function of the temperature dependence of n_2. Most likely n_2 is a decreasing function of the temperature, to vanish for temperatures above which the chemical bond is broken. This temperature, playing the role of a critical temperature, should therefore be of the order of a few 10^3 K. The nonlinear optical property resulting in self-focusing can be interpreted as an attractive force acting between the photons. If the photon gas is dense enough it can undergo Bose-Einstein condensation, and if the attractive force is strong enough, it is conceivable that it becomes superfluid, by undergoing a second order phase transition.

A photon gas obeying a Planck black body radiation law already is a degenerate Bose-Einstein gas. The same must be true even more for the low temperature photon gas of a laser beam.

In a refractive medium the photons can be understood as quasiparticles having an effective mass m^*

and moving with the velocity $v = c/n$. The wave length in the refractive medium is $\lambda^* = \lambda/n$, and one has

$$\lambda^* = h/m^* \, v \,. \tag{3}$$

With $\lambda = h/m \, c$, $v = c/n$ we obtain from (3) an expression for m^*:

$$m^* = n^2 \, m \,. \tag{4}$$

From m^* one can compute the rest mass m_0^* of these quasiparticles:

$$m_0^* = m^* \sqrt{1 - v^2/c^2} = n \sqrt{n^2 - 1} \, m \,, \tag{5}$$

which shows that $m_0^* = 0$ for $n = 1$.

For $n \gg 1$ one has $m^* \cong m_0^*$, and the Bose gas of the quasiparticles of mass m^* is nonrelativistic. Under these conditions Bose-Einstein condensation occurs if the temperature T is less than a critical temperature T_B, given as follows [2]:

$$kT < kT_B \cong (\pi \, \hbar^2/m^*) \, N^{2/3}, \tag{6}$$

where N is the number density of the quasiparticles. Putting

$$(3/2) \, kT = (m^*/2) \, v^2 \tag{7}$$

one finds from (6) that

$$N > 8 \, (m^* \, v/h)^3 \,. \tag{8}$$

With $m^* = n^2 \, m$, $v = c/n$ this is

$$N > 8 \, (n/\lambda)^3 = 8/\lambda^{*3} \,. \tag{9}$$

For Bose-Einstein condensation to occur, the beam intensity I must therefore be larger than a critical intensity I_c given by

$$I_c = 8 \, (n/\lambda)^3 \, (c/n) \, h \nu = 8 \, h \, c^2 \, n^2/\lambda^4 \,. \tag{10}$$

Then, if

$$P/\pi \, r^2 \geqq I_c \,, \tag{11}$$

where r is the beam radius, a transition into a superfluid state may occur. From (11) one obtains for $P = P_c$

a critical beam radius r_c below which the transition would take place:

$$r < r_c = \frac{\lambda^3}{(4\pi)^2 \sqrt{\hbar c}} \frac{1}{n_2^{1/2} n} .$$ (12)

If $P > P_c$, less focussing is needed and one there finds that

$$r < r_c \sqrt{P/P_c} .$$ (13)

For the above example one has

$$r_c \cong 0.4/n \text{ [cm]} .$$ (14)

Very large effective refractive indices are possible, up to $n \cong 10^3$, under conditions involving self-induced transparency [1]. To obtain for the given example under these conditions a superfluid beam with a radius of $r \cong 0.5$ cm, would according to (13) require to make $P/P_c \cong 10^6$, that is $P \sim 2 \times 10^{10}$ Watt.

The transition temperature T_B in the gas of quasi-particles is according to (6) and (7) ($m^* = n^2 m$ and $v = c/n$) given by

$$k T_B = (1/3) m^* v^2 = (1/3) m c^2 = (1/3) h v$$ (15)

and therefore is unchanged if compared with a free photon gas. However, a comparison of the uncertainty principle for the quasiparticles and the free photons shows, why the quasiparticles have a much better chance to undergo a phase transition into a superfluid state. For a free photon the uncertainty principle is

$$m r c \gtrsim \hbar ,$$ (16)

whereas for a quasiparticle it is

$$m^* r v \gtrsim \hbar .$$ (17)

With $m^* = n^2 m$ and $0 = c/n$ it can be brought into the form

$$m r c \gtrsim \hbar/n ,$$ (18)

which shows that, provided $n \gg 1$, the quasiparticles can be much more densely packed than the free photons, greatly enhancing the chance for a second order phase transition.

There is no obvious reason why the described conditions to get Bose-Einstein condensation, followed by a second order phase transition into a superfluid state, cannot be met. In a superfluid laser beam all the photons would be highly correlated, a property which

would find its establishment in the formation of an energy gap. As a result, individual photons of the superfluid condensate would not be scattered out of the beam. To speak of photons moving through a refractive medium is a not quite correct, albeit convenient way to describe the real physical situation. In reality, part of the energy is stored in excited electronic states of the atoms or molecules making up the refractive medium. Because the photon number density in the medium is $\sim (n/\lambda)^3$, the energy density is larger by the factor n^3 than in vacuum. The fraction of the energy stored in the atoms therefore is

$$f = (n^3 - 1)/n^3 = 1 - 1/n^3 .$$ (19)

The view, that the energy is stored in excited electronic states, is also supported by the mass of the quasi-particles. Expressed through the free photon mass $m = h v/c^2$, this mass is

$$m^* = n^2 m = n^2 (h v/c^2) .$$ (20)

Typically $h v \sim 1$ eV $\sim 10^{-12}$ erg, and one finds that $m \sim 10^{-33}$ g and $m^* \sim 10^{-27}$ g. The mass of the quasi-particles is therefore of the same order of magnitude as the electron mass.

The finite rest mass of the quasiparticles leads to a finite range of interaction and which is given by the Compton wave length:

$$\lambda_c = \frac{h}{m_0^* c} = \frac{h}{mc} \frac{1}{n\sqrt{n^2 - 1}} = \frac{\lambda}{n\sqrt{n^2 - 1}} .$$ (21)

To have for the quasiparticles Bose-Einstein statistics means that the excited atoms or molecules of the refractive medium must have an integer spin. This raises the interesting question, if there could be a connection between such a superfluid beam and the recently discovered high temperature superconductors. As it appears, these superconductors cannot be explained by the mechanism of the BCS theory. As a possible alternative mechanism it has been suggested that bound spin-0 electron pairs are formed in position space, perhaps in single atoms. A superfluid photon beam, if it can be realized, may work by a similar mechanism. The propagation without resistance of a superfluid photon beam in a refractive medium could therefore be called optical superconductivity or perhaps supertransparency.

[1] A. Yariv, Quantum Electronics, John Wiley & Sons, New York 1975, pp. 498 ff., and 403.

[2] D. R. Tilley and J. Tilley, Superfluidity and Superconductivity, John Wiley & Sons, New York 1974, p. 26 ff.